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EXPLOSIVE DISCHARGE OF A GAS-SATURATED LIQUID FROM CHANNELS AND TANKS

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A mathematical model for the discharge of a gas-saturated liquid from cylindrical channels is developed. Two limiting cases of linear and quadratic relations between the flow friction force and the flow velocity are considered. It is established that the process of evacuation from a semi-infinite channel consists of two stages. In the initial stage, the flow drag can be ignored, and the process of discharge is described by a Riemann wave solution. For the next stage, in which inertia is insignificant, nonlinear equations are obtained and self-similar solutions are constructed for them. The problem of flow through a slot in a tank of finite volume is solved. It is shown that the discharge proceeds either in a gas-dynamic choking regime or in a subsonic regime, depending on the conditions inside the tank and at the outlet. Examples of numerical calculations are given.

Introduction. We consider a gas-saturated liquid flow at certain pressure p_0. A decrease in the flow pressure to values p_0 leads to "boiling" of the liquid (formation of a gas phase). In constructing a theoretical model for flow with gas evolution, we adopt the following assumptions. The gas phase is produced only by evolution of the dissolved gas (the liquid is considered "cold," and, hence, the partial pressure of the liquid vapors in the gas phase can be ignored). The relation between the current concentration of the dissolved gas and the pressure obeys the Henry law, and, hence, the evolution of the dissolved gas proceeds in an equilibrium regime. This regime can occur when the liquid contains a rather great quantity of additive particles, which are centers of gas evolution. In particular, for equilibrium gas evolution, it is necessary that the characteristic time of diffusion $t_D = 1/n^{2/3}D$ (n is the number of additive particles and D is the diffusivity in the liquid) be much smaller than the characteristic time of the problem $\tilde{t}$ ($t_D \ll \tilde{t}$). Hence, for n, we obtain the estimate $n \gg \tilde{n}$, where $\tilde{n} = (D\tilde{t})^{3/2}$. In addition, under the above condition of equilibrium gas evolution, the capillary forces at the interface are also insignificant. For this, in turn, the radii of gas inclusions should obey the relation $a \gg \tilde{a}$, where $\tilde{a} = 2\sigma/p$ (σ is the surface-tension coefficient and p is the pressure). We assume that the velocities of the phases coincide and the temperature of the system is constant and equal to the initial temperature T_0.

A qualitatively similar pattern takes place for flow of an ordinary boiling liquid where the flow pressure reaches the saturation value p_0 that corresponds to the initial temperature of the liquid T_0 [$p_0 = p_S(T_0)$]. In this case, too it is possible to construct a similar theory with equilibrium phase transitions under adiabatic conditions, using the Clausius–Clapeyron equation instead of the Henry law (this case is covered in a separate paper).

1. Basic Equations. Using the adopted assumptions, we write the following equations of mass and momenta for a constant-area channel:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho w)}{\partial z} = 0; \qquad (1.1)$$

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$$\rho\left(\frac{\partial w}{\partial t}+w\frac{\partial w}{\partial z}\right)=-\frac{\partial p}{\partial z}-\tau. \tag{1.2}$$

Here ρ, w, and p are the mean density of the gas–liquid mixture, the velocity, and the pressure, and τ is the reduced viscous friction force. For the mean density, we can write

$$\rho=\rho_l^0(1-\alpha_g)+\rho_g^0\alpha_g, \tag{1.3}$$

where ρ_i^0 $(i=l,g)$ are the true densities of the liquid and gas in the free state, the subscripts l and g refer to the liquid and gas, respectively, and α_g is the volumetric content of the gas phase.

Below, we assume that the liquid is incompressible and the gas is calorically perfect:

$$\rho_l^0=\mathrm{const},\quad p=\rho_g^0RT \tag{1.4}$$

(R is the gas constant).

According to the Henry law, for the mass concentration of the gas dissolved in the liquid we have

$$k=k_0p/p_0, \tag{1.5}$$

where k_0 is the mass concentration of the saturated gas at pressure p_0. Then, for the mean density of the gas, we obtain

$$\rho_g=\rho_l^0k(1-\alpha_g)+\rho_g^0\alpha_g. \tag{1.6}$$

In addition, by virtue of the assumption of equilibrium for the velocities, we have $\rho_g/\rho=k_0$. Hence, relations (1.3)–(1.6) lead to the following equation of state for the gas–liquid system considered:

$$\frac{1}{\rho}=\left[\frac{1}{\rho_l^0}-\frac{k_0}{\rho_{g0}^0(1-k_0)}\left(1-\frac{p_0}{p}\right)\right]\left[1-\frac{k_0}{1-k_0}\left(\frac{p}{p_0}-1\right)\right]^{-1}. \tag{1.7}$$

Here ρ_{g0}^0 is the true density of the free gas at pressure p_0 and temperature T_0. Thus, the gas-saturated liquid in some cases can be considered as a barotropic medium with the equation of state (1.7). In most cases, $k_0\ll 1$, and, hence, expression (1.7) can be written as

$$\frac{1}{\rho}=\frac{1}{\rho_l^0}-\frac{k_0}{\rho_{g0}^0}\left(1-\frac{p_0}{p}\right). \tag{1.8}$$

We note that for this equation of state, rarefaction shock waves are impossible. In the case where the mean density of the gas dissolved in the liquid $\rho_l^0k_0$ is close to the true density of the gas ρ_{g0}^0 in the free state $(\rho_l^0k_0\approx\rho_{g0}^0)$, the gas–liquid system is called perfect and Eq. (1.8) is simplified and written as

$$\rho=\frac{\rho_g^0}{k_0}\frac{p}{p_0}. \tag{1.9}$$

This case refers to water saturated with carbon dioxide, for example, at $T=288$ K [1].

Using (1.8), for the velocity of sound we have

$$C^2=\frac{\rho_{g0}^0}{k_0p_0}\frac{p^2}{\rho^2}. \tag{1.10}$$

When (1.9) is valid, the velocity of sound is given by $C^2=k_0p_0/\rho_{g0}^0$. Hence, for the perfect system, the velocity of sound is constant.

For comparison, we give the equation of state of a gas–liquid system in which the gas is entirely in the free state:

$$\frac{1}{\rho}=\frac{1-k_0}{\rho_l^0}+\frac{k_0}{\rho_{g0}^0}\frac{p_0}{p}.$$

Here k_0 is the mass gas content in the two-phase mixture.

To specify the viscous friction force τ, we consider two limiting cases. The first case is a "thin" channel with a linear relation between the reduced viscous friction force and the flow velocity:

$$\tau = \rho \frac{w}{t_{(w)}}. \tag{1.11}$$

For laminar flow in a cylindrical channel with a low volumetric concentration of the gas phase (the carrier phase is the liquid) the characteristic parameter $t_{(w)}$ in the friction law (1.11) can be approximated by $t_{(w)} = a^2/8\nu_l$, where a is the radius of the channel and ν_l is the kinematic viscosity.

The second case is a "thick" channel with turbulent flow and a quadratic drag law [2]:

$$\tau = \rho \frac{|w|w}{z_{(w)}}, \quad z_{(w)} = \frac{2a}{\lambda}, \quad \lambda = \left(2\log\frac{a}{\delta} + 1.74\right)^{-1}, \tag{1.12}$$

where δ is the roughness. We note that within the framework of the assumptions adopted here, the system considered is an incompressible liquid (ρ_0 = const and $C = \infty$) at pressures exceeding the saturation pressure ($p > p_0$).

2. Evacuation from the Channels. Using the equation of momentum (1.2) with allowance for (1.7) and (1.8), it is possible to estimate the characteristic times t_* and distances z_* for which inertia effects can affect flow pattern in the channel. For compressible fluid flow in channels, the maximum (critical) flow velocities are limited by the local velocity of sound, and, therefore, in the estimations, the velocity of sound should be adopted as the characteristic jump of velocities. In the case of the linear friction law (1.11), assuming that, in Eq. (1.2), the terms due to the inertia effects $\rho(\partial w/\partial t)$ and $\rho(\partial(w^2/2)/\partial z)$ and the terms due to the flow drag $\rho w/t_{(w)}$ satisfy the conditions $\rho C/t_* \sim \rho C/t_{(w)}$ and $\rho C^2/2z_* \sim \rho C/t_{(w)}$, we obtain the following estimates for the characteristic times and distances: $t_* \sim t_{(w)}$ and $z_* \sim Ct_{(w)}/2$.

Hence, if in the system considered disturbances arise with characteristic times and distances far exceeding t_* and z_*, the inertia effects do not have a significant effect on the subsequent flow pattern. If, for example, the problem of sudden depressurization at the end of the channel is considered, the inertia effects are significant mainly in the initial stage before the times t_* and on the end segment of length of order z_*.

In the case of the quadratic drag law (1.12), we obtain the estimates $t_* \sim z_{(w)}/C$ and $z_* \sim z_{(w)}$.

Let us consider the case where the inertia terms in the equation of momenta are negligible. In the problem of depressurization of a channel, this case corresponds to the later stage ($t \gg t_*$) of evacuation. Then, using the continuity and momenta equations and ignoring the terms on the left side of Eq. (1.2), for the cases of the linear and quadratic friction laws we obtain

$$\frac{\partial p}{\partial t} = C^2 t_{(w)} \frac{\partial^2 p}{\partial z^2}, \quad w = -\frac{t_{(w)}}{\rho}\frac{\partial p}{\partial z}; \tag{2.1}$$

$$\frac{\partial p}{\partial t} = C^2 \sqrt{2z_{(w)}}\,\mathrm{sgn}\left(\frac{\partial p}{\partial z}\right)\frac{\partial}{\partial z}\sqrt{\rho\left|\frac{\partial p}{\partial z}\right|}, \quad w|w| = -\frac{z_{(w)}}{\rho}\frac{\partial p}{\partial z}. \tag{2.2}$$

Using Eqs. (2.1) and (2.2) we consider the problem of sudden pressure release to a value $p_e < p_0$. Let the initial velocity of the channel flow be equal to zero. Then, the corresponding boundary and initial conditions for Eqs. (2.1) and (2.2) are written as

$$p = p_0 \quad (t = 0,\ z \geqslant 0), \qquad p = p_e \quad (t > 0,\ z = 0). \tag{2.3}$$

This problem is self-similar. For convenience, we bring Eqs. (2.1) to the form

$$\frac{\partial P}{\partial t} = \bar{C}^{-2}k^{(1)}\frac{\partial^2 P}{\partial z^2}, \quad P = \frac{p}{p_0}, \quad \bar{C} = \frac{C}{C_0}, \quad k^{(1)} = C_0^2 t_{(w)},$$
$$\bar{C}^2 = \frac{P^2}{\mathcal{R}^2}, \quad \mathcal{R} = \left((1-\mathcal{R}_*) + \frac{\mathcal{R}_*}{P}\right)^{-1}, \quad C_0 = \sqrt{\frac{p_0}{\rho_l^0\mathcal{R}_*}}, \quad \mathcal{R}_* = \frac{\rho_l^0 k_0}{\rho_{g0}^0}, \quad \mathcal{R} = \frac{\rho}{\rho_l^0}. \tag{2.4}$$

Here the dimensionless parameter $\mathcal{R}_*$ corresponds to the Ostwald absorption coefficient [1]. A solution of this equation subject to conditions (2.3) can be sought in the form $P = P(\xi)$, where $\xi = z/\sqrt{k^{(1)}t}$. In this case,

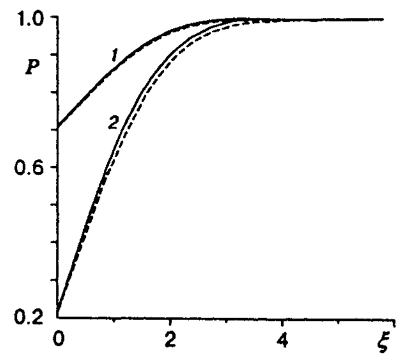

Fig. 1

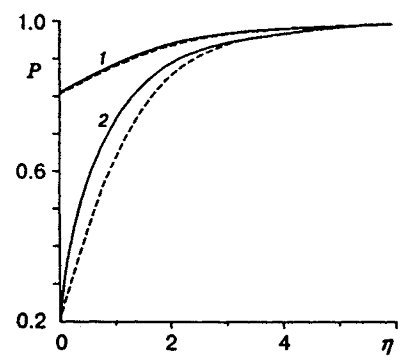

Fig. 2

conditions (2.3) lead to $P(0) = P_e$ and $P(\infty) = 1$ ($P_e = p_e/p_0$). Then, Eq. (2.4) in the self-similar variables is written as

$$-\frac{\xi}{2}\frac{dP}{d\xi} = \bar{C}^2 \frac{d^2P}{d\xi^2}. \tag{2.5}$$

For the rate of mass flow through the open end of the channel $q = -(\rho w)_0$, we have

$$q = \frac{t_{(w)} p_0}{\sqrt{k^{(1)} t}} P'(0). \tag{2.6}$$

In the case of slight pressure drops ($p - p_0 \ll p_0$), setting $\bar{C}^2 \approx 1$, we can linearize Eq. (2.5). Then its solution has the form [3]

$$P = P_e + (1 - P_e)\hat{O}(\xi), \quad \hat{O}(\xi) = \frac{2}{\sqrt{\pi}} \int\limits_0^{\xi/2} e^{-\alpha^2}\, d\alpha. \tag{2.7}$$

Figure 1 shows the dimensionless pressure distribution along the channel length in the self-similar variable for $z = 0$ and $\mathcal{R}_* = 1.74$ for two pressures at the boundary of the channel [solid curves 1 and 2 correspond to $P(\xi = 0) = 0.2$ and 0.7, and the dashed curves are calculated from the analytical solution (2.7)].

For the mass flow rate defined by expression (2.6), we obtain $q_* = t_{(w)}(p_0 - p_e)/\sqrt{\pi k^{(1)} t}$. The coefficient $\chi^{(1)} = q/q_* = \sqrt{\pi} P'(0)/(1 - P_e)$ defines the correction to the flow rate due to the nonlinearity of Eq. (2.5).

In the case of the quadratic friction law, from Eq. (2.2) we have

$$\frac{\partial P}{\partial t} = \bar{C}^2 k^{(2)} \frac{\partial}{\partial z}\sqrt{\mathcal{R}\frac{\partial P}{\partial z}}, \quad \mathcal{R} = \frac{\rho}{\rho_l^0}, \quad k^{(2)} = C_0^2 \sqrt{\frac{2 z_{(w)} \rho_l^0}{p_0}}.$$

This equation is written in the self-similar variables as

$$-\frac{2}{3}\eta \frac{dP}{d\eta} = \bar{C}^2 \frac{d}{d\eta}\sqrt{\mathcal{R}\frac{dP}{d\eta}}, \quad \eta = \frac{z}{(k^{(2)} t)^{2/3}}. \tag{2.8}$$

Introducing the new parameters $\tilde{P} = (P - P_e)/(1 - P_e)$ and $\tilde{\eta} = (1 - P_e)^{1/3}\eta$ and using Eq. (2.8), for slight pressure jumps ($\bar{C} \approx 1$ and $\mathcal{R} \approx 1$) we obtain

$$-\frac{2}{3}\tilde{\eta}\frac{d\tilde{P}}{d\tilde{\eta}} = \frac{d}{d\tilde{\eta}}\sqrt{\frac{d\tilde{P}}{d\tilde{\eta}}}. \tag{2.9}$$

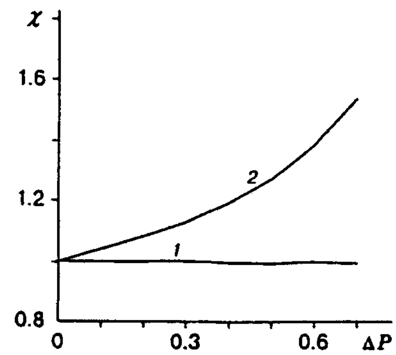

Fig. 3

Conditions (2.3) for these variables are written as

$$\tilde{P}(0) = 0, \qquad \tilde{P}(\infty) = 1. \tag{2.10}$$

A solution of Eq. (2.9) subject to (2.10) has the form

$$\tilde{P} = \frac{9}{2\alpha^2}\left(\frac{\tilde{\eta}}{\tilde{\eta}^2 + \alpha^2} + \frac{1}{\alpha}\arctan\frac{\tilde{\eta}}{\alpha}\right), \quad \alpha = \left(\frac{9\pi}{4}\right)^{1/3}. \tag{2.11}$$

Figure 2 shows the pressure distribution calculated from Eq. (2.8) for $\mathcal{R}_* = 1.7$ [curves 1 and 2 correspond to $P(\eta = 0) = 0.8$ and 0.2, respectively, and the dashed curves correspond to solution (2.11)].

According to the formula of the rate of flow of the mixture through a unit cross-sectional area of the channel

$$q = -(\rho w)_0 = \sqrt{z_{(w)}\left(\rho\frac{\partial p}{\partial z}\right)_0},$$

we have $q = \sqrt{z_{(w)}\rho(p_e)p_0 P'(0)}/(k^{(2)}t)^{1/3}$. In the case of slight pressure jumps, with allowance for (2.11), this leads to

$$q_* = \frac{\sqrt{z_{(w)}\rho(p_e)(1 - P_e)^{4/3}p_0\tilde{P}'(0)}}{(k^{(2)}t)^{1/3}}, \quad \tilde{P}'(0) = \frac{4}{\pi}\left(\frac{4}{9\pi}\right)^{1/3}.$$

Similarly, for the dimensionless relation $\chi^{(2)}$ we can write

$$\chi^{(2)} = \sqrt{P'(0)/(1 - P_e)^{4/3}\tilde{P}'(0)}.$$

Figure 3 shows curves of $\chi^{(i)}$ versus ΔP for $\mathcal{R}_* = 1.7$. The numbering of curves 1 and 2 corresponds to the values of i.

For the initial stage ($t \ll t_*$) of evacuation, ignoring the terms due to flow friction in Eq. (1.2), we have a situation similar to the gas-dynamic problem where the discharge process is described by a simple wave solution. In this case, the Riemann integral holds:

$$w(p) = \int_{p_0}^{p} \frac{dp}{\rho C}.$$

In the case of a gas-saturated liquid, ρ and C are defined by relations (1.8) and (1.10). From the Riemann integral we obtain $w(p) = \sqrt{k_0 p_0/\rho_{g0}^0}\,\ln(p/p_0)$.

For slight pressure jumps ($p_0 - p \ll p_0$), assuming that $C(p) = C(p_0) = C_0$ and $\rho(p) \approx \rho(p_0) = \rho_l^0$, we obtain $w(p) = (p - p_0)/(\rho_l^0 C_0)$.

Here two discharge regimes [4] are possible. The first occurs when $-w(p_e) < C(p_e)$ and rarefaction to pressure p_e is spread in the tube at velocity $w(p_e) + C(p_e)$, and at the outlet ($z = 0$) the pressure p_e is established. In the case where $-w(p_e) > C(p_e)$, rarefaction to the value p_e in the tube is impossible, since the corresponding disturbance is carried by the medium toward $z < 0$ and at the channel outlet, the critical pressure p_C is established, which leads to $-w(p_C) = C(p_C)$. With allowance for (1.8) and (1.10), the Riemann integral leads to the following transcendental equation for p_C:

$$C(p_C) = \int_{p_C}^{p_0} \frac{dp}{\rho(p)C(p)}.$$

Replacing $\rho(p)$ and $C(p)$ in the right side by expressions (1.8) and (1.10), we obtain $(1 + \ln P_C)/P_C = (\mathcal{R}_* - 1)/\mathcal{R}_*$ and $P_C = p_C/p_0$. In the case of the perfect system, this leads to $p_C = p_0/e$. We note that the leading edge of the rarefaction wave moves relative to the channel with velocity $C_0 = C(p_0)$.

3. Steady Flow through Cylindrical Channels. For steady flows, Eqs. (1.1) and (1.2) can be written as

$$\rho w = m = \text{const}; \tag{3.1}$$

$$m\frac{dw}{dz} + \frac{dp}{dz} = -\tau. \tag{3.2}$$

In the case of the linear friction law (1.7), from Eqs. (3.1) and (3.2) we have

$$\left(1 - \frac{m^2}{\rho^2 C^2}\right)\frac{dp}{dz} = -\frac{m}{t_{(w)}}.$$

Hence, integrating over the channel length (from zero to z_c), we obtain

$$\int_{p_+}^{p_-} \left(1 - \frac{m^2}{\rho^2 C^2}\right) dp = -\frac{m z_c}{t_{(w)}}. \tag{3.3}$$

The subscripts "plus" and "minus" denote that the parameter refers, respectively to the channel inlet ($z = 0$) and outlet ($z = z_c$). Using (1.11), we bring Eq. (3.3) to the form

$$p_+ - p_- - m^2\left(\frac{1}{\rho_+} - \frac{1}{\rho_-}\right) = \frac{m z_{(w)}}{t_{(w)}}.$$

Solving this equation for m, we obtain

$$m = \left[\frac{z_c}{2t_{(w)}} - \sqrt{\frac{z_c^2}{4t_{(w)}^2} + (p_+ - p_-)\left(\frac{1}{\rho_+} - \frac{1}{\rho_-}\right)}\right]\left(\frac{1}{\rho_+} - \frac{1}{\rho_-}\right)^{-1}.$$

Substituting the pressure–density relations from (1.8), we have

$$m = \frac{\rho_{g0}^0}{k_0 p_0}\left[\frac{z_c}{2t_{(w)}} - \sqrt{\left(\frac{z_c}{2t_{(w)}}\right)^2 - \frac{(p_+ - p_-)^2}{p_+ p_-}\frac{k_0 p_0}{\rho_{g0}^0}}\right]\left(\frac{1}{p_+} - \frac{1}{p_-}\right)^{-1}. \tag{3.4}$$

This formula expresses the rate of flow of the boiling liquid through the cylindrical channel in terms of the known pressures at the channel inlet and outlet.

In the case of the quadratic friction law, this relation has the form

$$m = \sqrt{\left(\ln\frac{\rho_-}{\rho_+} + \frac{z_c}{z_{(w)}}\right)^{-1} \int_{p_+}^{p_-} \rho\, dp}.$$

Hence, with allowance for (1.8) we have

$$m = \left(p_+ - p_- - \frac{k_0 p_0}{\rho_{g0}^0}\left(\frac{1}{\rho_l^0} - \frac{k_0}{\rho_{g0}^0}\right)\ln\frac{p_+\rho_-}{p_-\rho_+}\right)^{1/2}\left(\left(\frac{1}{\rho_l^0} - \frac{k_0}{\rho_{g0}^0}\right)\left(\ln\frac{\rho_+}{\rho_-} + \frac{z_c}{z_{(w)}}\right)\right)^{-1/2}. \tag{3.5}$$

For the perfect system, we have

$$m = \sqrt{\frac{\rho_{g0}^0}{2k_0 p_0}} \sqrt{\frac{p_+^2 - p_-^2}{\ln(p_+/p_-) + z_c/z_{(w)}}}.$$

For discharge from a large tank, we can assume that the pressure and velocity at the inlet of the tubular nozzle are related by the Bernoulli integral:

$$\frac{w_+^2}{2} + \int_{p_0}^{p_+} \frac{dp}{\rho} = 0, \quad m = \rho_+ w_+ \quad \text{or} \quad \frac{m^2}{2\rho_+^2} + \int_{p_0}^{p_+} \frac{dp}{\rho} = 0.$$

Then, using the equations of state (1.8), we obtain

$$\frac{m^2}{2\rho_+^2} + \left(\frac{1}{\rho_l^0} - \frac{k_0}{\rho_{g0}^0}\right)(p_+ - p_-) + \frac{k_0 p_0}{\rho_{g0}^0} \ln \frac{p_+}{p_0} = 0. \tag{3.6}$$

Here p_0 is the pressure in the volume far from the outlet cross section of the tank. Thus, to determine the rate of flow of the gas-saturated liquid through the tube, it is necessary to consider Eqs. (3.6) and (3.4) [or (3.5)] simultaneously. For a slight pressure jump ($p_0 - p_+ \ll p_0$), instead of (3.6) we can write

$$\frac{w_+^2}{2} + \frac{p_+ - p_0}{\rho_l^0} = 0 \quad \text{or} \quad m = \sqrt{2(p_0 - p_+)\rho_l^0}.$$

Relations (3.4) [or (3.5)] together with (3.6) define the rate of flow of the gas-saturated liquid from a large tank provided that the pressure at the channel outlet p_- is higher than the pressure p_C at which flow choking occurs [$w_- = C(p_C)$]. If the pressure outside the channel is lower than p_C, we obtain the critical flow rate, defined by

$$m = \rho(p_C)C(p_C) = m_C. \tag{3.7}$$

Thus, depending on the gas-saturation conditions in the tank, the conditions outside the tank determined by the pressure, and the geometrical and hydraulic characteristics of the channel, there are two discharge regimes: critical and subcritical. In the case of subcritical flow ($p_- > p_C$ and $m < m_C$), using the known parameters of the liquid in the tank (for example, the temperature T_0) and the pressure p_- outside the tank, it is necessary to simultaneously solve Eqs. (3.5) and (3.6), which form a system of transcendental equations for the two unknown parameters m and p_+. For the critical efflux ($p_- < p_C$), the system of three equations (3.5), (3.6), and (3.7) for the three unknown parameters m_C, p_+, and p_C should be considered.

4. Discharge of Champagne. We consider the problem of evacuation through a slot in a tank of finite volume. We assume that the pressure in the main volume at sufficient distance from the slot is uniform (the homobaric condition), and the discharge is quasisteady. The equation of conservation of mass for the gas–liquid mixture located in volume V is written as

$$V \frac{d\rho_{(i)}}{dt} = -S\rho_{(e)}w_{(e)}, \tag{4.1}$$

where $\rho_{(i)}$ is the mean density of the mixture in the tank, $\rho_{(e)}$ and $w_{(e)}$ are the density and flow velocity at the exit from the slot, and S is the cross-sectional area of the slot. By virtue of the assumptions adopted above, the pressure in the volume and at the outlet section and the flow velocity are related by the Bernoulli integral

$$\frac{w_{(e)}^2}{2} + \int_{p_{(e)}}^{p_{(i)}} \frac{dp}{\rho} = 0. \tag{4.2}$$

In this case, two discharge regimes are possible. The first is the gas-dynamic choking regime, in which the flow velocity $w_{(e)}$ is equal to the local velocity of sound. In this case, the pressure p_C at the exit from the slot

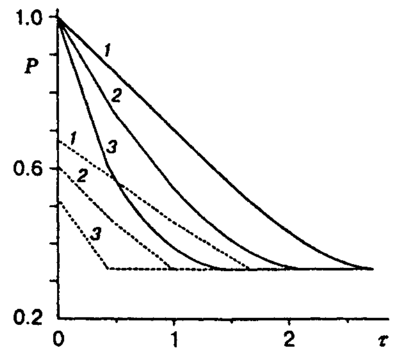

Fig. 4

is higher than the external pressure p_e and is defined by the equation

$$C(p_C) = w(p_C), \tag{4.3}$$

where $w(p_C) = w_{(e)}$ is determined from the Bernoulli integral (4.2).

Taking into account (1.8), (1.10), and (4.2), from (4.3) we obtain the following transcendental equation for determining p_C from the current pressure inside the tank $p_{(i)}$:

$$(1-\mathcal{R}_*)P_C + \mathcal{R}_* = \sqrt{2\mathcal{R}_*}[(1-\mathcal{R}_*)(P_{(i)} - P_C) + \mathcal{R}_* \ln(P_{(i)}/P_C)]^{1/2}, \quad P_{(i)} = \frac{p_{(i)}}{p_0}, \quad P_C = \frac{p_C}{p_0}. \tag{4.4}$$

If the value of p_C is higher than the external pressure $p_{(e)}$, instead of $\rho_{(e)}$ and $w_{(e)}$ in Eq. (4.1) it is necessary to use the density and velocity obtained from formulas (1.7) and (4.2) for $p = p_C$. Using (4.1) and taking into account (4.4), we obtain the following equation for pressure variation in the tank:

$$\mathcal{R}_*^{3/2}\frac{dP_{(i)}}{d\tau} = -P_C[(1-\mathcal{R}_*)P_{(i)} + \mathcal{R}_*]^2 \qquad \left(\tau = t\frac{S}{V}\sqrt{\frac{p_0}{\rho_e^0}}\right). \tag{4.5}$$

For the perfect system, relations (4.4) and (4.5) yield $dP_{(i)}/d\tau = -P_{(i)}/\sqrt{e}$ and $P_C = P_{(i)}/\sqrt{e}$.

If the pressure p_C (or P_C) determined from Eq. (4.4) for the initial state $[p_{(i)} = p_0]$ is not higher than the external pressure p_e ($p_C \leqslant p_e$), then, taking into account (1.8) and (4.2) for $p_{(e)} = p_e$ from (4.1), instead of Eq. (4.5) we obtain

$$\mathcal{R}_*\frac{dP_{(i)}}{d\tau} = -\sqrt{2}P_e[(1-\mathcal{R}_*)P_{(i)} + \mathcal{R}_*]^2[(1-\mathcal{R}_*)(P_{(i)} - P_e) + \mathcal{R}_* \ln(P_{(i)}/P_e)]/[(1-\mathcal{R}_*)P_e + \mathcal{R}_*]. \tag{4.6}$$

Hence, for the perfect system, we obtain the equation

$$\frac{dP_{(i)}}{d\tau} = -P_e\sqrt{2\ln\frac{P_{(i)}}{P_C}}.$$

Consequently, if the initial pressure is rather high $p_C > p_e$, the discharge process consists of two stages. In the first stage — from the beginning of discharge up to the moment when the critical pressure becomes equal to the external pressure ($p_C = p_e$) — the process is described by the system of two equations (4.4) and (4.5). At the second stage — from the moment when $p_C = p_e$ up to the moment when the pressure in the tank decreases to the external pressure p_e $[p_{(i)} = p_e]$ — the process is described by one equation (4.6).

In the case of a perfect system, the times of the first and second stages of the discharge are given by

$$\tau^{(1)} = -\sqrt{e}\ln(\sqrt{e}P_e), \qquad \tau^{(2)} = \int_{P_e}^{\sqrt{e}P_e}\frac{dP_{(i)}}{P_e\sqrt{2\ln(P_{(i)}/P_e)}}. \tag{4.7}$$

If, for the initial state, the condition $p_C \leqslant p_e$ is satisfied, the evacuation process consists of only the second stage. In the case of the perfect system, the time of evacuation is given by

$$\tau = \int\limits_{P_e}^{1} \frac{dP_{(i)}}{P_e\sqrt{2\ln\left(P_{(i)}/P_e\right)}}. \tag{4.8}$$

Figure 4 shows curves of the dimensionless pressures inside the tank (solid curves) and at the exit from the slot (dashed curves) versus the dimensionless time for $p_0 = 0.3$ MPa. Curves 1–3 are obtained for the values $\mathfrak{R}_* = 1.7$, 1.0, and 0.51, which correspond to the Ostwald coefficients for water with carbon dioxide at temperatures $T = 273$, 288, and 323 K. Analytical formulas (4.7) and (4.8) and the above numerical calculations show that for $V = 10^{-3}$ m^3, $S = 10^{-4}$ m^2, and $p_0 \sim 0.2$–0.5 MPa, the characteristic time of evacuation is one or two seconds. Then, from the estimates at the beginning of the article it follows that for the flow solution described, the number of additive species should be much greater than $\tilde{n} \approx 10^{14}$ m^{-3} and the radii of gas inclusions and the volumetric gas content $[\alpha_g = (4/3)\pi a^3 n]$ near the initial state should satisfy the conditions $a \gg 10^{-6}$ m and $\alpha_g \gg 10^{-4}$.

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